AM205: Assignment 0

Assignment 0 is for your own edification, it should provide some problems for you to refresh/test/hone your Matlab programming. This assignment will not be assessed — you do not need to submit your answers.

**Question 1**
Find the angle, \( \theta \), between the vectors, 
\[
 v_1 = (1.5, -2, 4, 10) \quad \text{and} \quad v_2 = (3.1, -1, 2, 2.5),
\]
using the formula 
\[
 \cos \theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|}.
\]

**Question 2**
Evaluate and plot the Chebyshev polynomial of degree 5 at 100 evenly spaced points in the interval \( x \in [-1, 1] \). Use the fact that we have the following recurrence relation for Chebyshev polynomials (where \( T_k \) denotes the Chebyshev polynomial of degree \( k \)):
\[
 T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x), \quad \text{for} \quad k \geq 2,
\]
and \( T_0(x) = 1, \ T_1(x) = x \).
Plot \( T_3(x)T_5(y) \) on a 100 \( \times \) 100 grid on the domain \( (x,y) \in [-1,1]^2 \). Try different plotting functions in Matlab for this grid-based data e.g. mesh, surf, contour.

**Question 3**
Use the iteration 
\[
 x_{k+1} = \frac{1}{2} \left( x_k + \frac{a}{x_k} \right).
\]
to approximate \( \sqrt{a} \). (This is known as Heron’s formula\(^1\) and in fact it is equivalent to Newton’s method for \( f(x) = x^2 - a \).) Choose an “initial guess” \( x_0 = a \) and iterate until \( |x_{k+1} - x_k| < \text{TOL} \). Determine the number of iterations required in the cases \( \text{TOL} = 10^{-3} \) and \( \text{TOL} = 10^{-9} \).

**Question 4**
Let \( f(x) = \tan(x) \), and consider the second order finite difference approximation
\[
 f_{\text{diff,2}}(x;h) \equiv \frac{f(x + h) - f(x - h)}{2h}.
\]
Plot the relative error in the \( f_{\text{diff,2}}(x;h) \) approximation at \( x = 1 \) as a function of \( h \) for \( h = 10^{-k}, k \in \{1, 1.5, 2, \ldots, 15.5, 16\} \). Make sure you use a “log-log” plot (use the command \text{loglog} in Matlab). Also, overlay dashed lines \( \alpha_1 h, \alpha_2 h^2, \alpha_3 / h \) to compare with the error rates you observe. (Choose \( \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R} \) to make the plot as aesthetically pleasing as possible.)

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\(^1\)Heron of Alexandria, 10–70 AD.
Question 5

\( y = \sin(x) \) is an analytic function, which means that the Taylor series

\[
x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots
\]

converges for any \( x \in \mathbb{R} \). Write a Matlab function (call it \texttt{sinTaylorSeries}) to evaluate this series; the function should take arguments \( x \) (a vector of values to evaluate the function at) and \( N \) (the number of terms in the series) and should return a vector \( y \) of function values and a vector \( \text{err} \) of (absolute) error values with respect to Matlab’s built-in \( \sin(x) \) function.

Use your function to plot \( y \) and \( |\text{err}| \) on the intervals \([-\pi, \pi]\) and \([-10\pi, 10\pi]\) for \( N = 10 \) and \( N = 100 \). (Use a “semilog-y” plot for \( |\text{err}| \), i.e. \texttt{semilogy} in Matlab.)